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BARK TAPER MODELS FOR CALIFORNIA CONIFERS

by

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Abstract

Upper-stem tree measurements are frequently made outside bark with the only bark thickness measurements being made at breast height. The current study develops equations and coefficients for estimating the bark thickness in both the upper stem and at the stumps based upon the bark thickness at breast height, the size of the tree (DBH and total height), and the height to the measurement in question.

Previously-published bark taper models were examined; the model adapted here is a previously-published hyperbolic ratio, with an adjustment. The adjustment is made to account for the more rapid taper on trees with thicker bark.

The data used for fitting and testing the taper models consisted of measurements on over 3,000 conifer trees measured by members of the Northern California Forest Yield Cooperative and the USDA Forest Service. The data set was split into two halves, one half for fitting and the other half for testing.

The adjusted hyperbolic model outperformed the other upper stem bark taper models for all species except red fir, where it gave similar results as one of the hyperbolic ratios and one of the segmented-polynormal models. For the bark taper at the stumps, the recommended power function outperformed the other models for all six conifer species examined.

While we are usually interested in the tree diameter inside bark for estimating tree volume, it is usually outside-bark diameters that are observed, often using an optical device to estimate tree diameters at various heights above the ground. This leaves one with the problem of estimating the bark thickness for each outside-bark diameter measured along the stem. The objective here is to examine alternative models for expressing the relative taper of the bark so that bark thickness

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observed at breast height can be used to estimate the bark thickness at various points along the stem.

The current study is a cooperative effort between the University of California, the USDA Forest Service Region 5 and the Pacific Southwest Research Station, the California Department of Forestry and Fire Protection, Michigan-California Lumber Company, and Crane Mills. The conifer species considered include the following:

Code	Species
PP	ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.)
SP	sugar pine (*P. lambertiana* Dougl.)
DF	Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco)
WF	white fir (*Abies concolor* (Gord. & Glend.) Lindl. ex Hildebr.)
RF	red fir (*A. magnifica* A. Murr.)
IC	incense-cedar (*Calocedrus decurrens* [Torr.] Florin)

For many of the same conifer species considered here, Khan, Bell, and Berg (1977), Richie and Hann (1984), Dolph (1984, 1989) and Larsen and Hann (1985) developed expressions for inside-bark diameter as a function of diameter outside bark. These include linear and nonlinear regressions and ratio models. However, as noted by Assman (1970, p.73), within the same species bark thickness varies "according to site, age, and racial characteristics of the tree". Thus, improvements in the predictions can be expected with more "local" information on the bark thickness of each tree. Also, the "true" relationship between inside- and outside-bark diameters is the subject of the current paper.

When measuring standing trees with an optical dendrometer, the usual process is to measure the double bark thickness at breast height (DBTBH) by either cutting into the stem with an axe or by using a bark gauge at 4.5 feet above the ground. Because such measurements are made with some error and since bark thickness varies around the stem, typically two such measurements are taken at right angles to each other and added to estimate the DBTBH, still with some error. Bark taper models can then be used to express the reduction in bark thickness as one goes from breast height to the tip or to the stump of the tree. Thus, the measurement of bark thickness at breast height "localizes" the predictions of bark thickness at other points. Below breast height one of two relationships are evident. Undamaged trees have bark thickness increasing from breast height down to the stump while trees that show damage due to fire and/or logging may have thinner bark at the stump than at breast height.

MODELS CONSIDERED

In this study, available bark thickness models were evaluated for their ability to predict the bark taper from felled tree data collected in five different studies of stem taper in California conifers (see "Data" below). The upper stem (above breast height) bark taper models evaluated are presented in Tables 1 and 2 while the lower stem (below breast height) models are considered in a later section. Table 1 includes basic hyperbolic models and Table 2 includes segmented polynomial models. Each equation is formulated to predict the relative bark thickness at a given height and the variables used include the following:

DBH	Diameter at breast height (4.5 feet)
DBT	Double bark thickness at height h
DBTBH	Double bark thickness at breast height
DIB	Diameter inside bark at height h
DOB	Diameter outside bark at height h
h	Height to measurement
THT	Total height of tree

We assume that the bark thickness at breast height can be measured and the variable predicted is the relative bark thickness, $\frac{DBT}{DBTBH}$. That is, Y_i is the estimated relative bark thickness, $\frac{DBT}{DBTBH}$, for bark taper model i ($i = 1, 2, \cdots, 10$).

Hyperbolic ratios (upper stem)

In his STX computer program for reducing measurements from optical dendrometers to tree diameter and volume estimates, Grosenbaugh (1974) provides three bark taper options. These ratios, illustrated in Figure 1, include a constant ratio, option 1, and two hyperbolic ratios, options 2 and 3[2]. The STX bark options, denoted as G1, G2, and G3, respectively, are as follows:

$$\text{(G1)} \qquad \frac{DOB - \widehat{DBT}}{DOB} = \frac{DBH - DBTBH}{DBH}$$

$$\text{(G2)}^{3} \qquad \frac{\widehat{DBT}}{DOB} = \left(\frac{DBTBH}{DBH}\right)\left(\frac{1}{2 - \frac{DOB}{DBH}}\right)$$

$$\text{(G3)} \qquad \frac{DOB - \widehat{DBT}}{DOB} = \left(\frac{DBH - DBTBH}{DBH}\right)\left(\frac{9}{10 - \frac{DOB}{DBH}}\right)$$

Conifer relative bark thickness decreases faster than the diameter ratio as illustrated in Figure 2 by the sample white fir data set. This causes a distinct lack of fit for bark options 1 and 3 which predict the bark ratio to decrease proportional to the diameter ratio and slower than the diameter ratio, respectively. Bark option 2 has the general form of the bark taper relationship, but, as illustrated in Figure 2, option 2 appears to underestimate the bark thickness in the top half of the tree and overestimate in the bottom half. Nonetheless, since it has less bias than either option 1 or option 3, option 2 has been used in previous stem taper studies (Wensel 1977 and Wensel and Krumland 1983). Thus, while Grosenbaugh's options 1 and 3 are dropped from further consideration, option 2 will be the first bark model considered here (see Table 1).

Models 2 through 5 (Table 1) are variants on this basic hyperbola (Brickell, 1970) restricted so that Y = 1 at breast height and Y = 0 at the tip where DOB = 0. Model 2 is similar to model 1 but the value of the coefficient is estimated from the data. Models 3 through 5 are variations of model 2 with different numbers of coefficients fitted to the various components.

Models 1 through 5 all showed differential bias associated with the bark thickness at breast height, even for the 4 parameter model 3 for which the residuals of upper stem bark thickness

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2 A fourth option is also provided for users to supply their own bark model.

3 Grosenbaugh (1974) denotes the two coefficients 1 and 2, by QUAN and DENO, respectively. However, if DBT is to equal DBTBH when DOB equals DBH, QUAN must equal DENO - 1 leaving option 2 with only one effective coefficient.

predictions consistently showed a negative correlation with the bark thickness at breast height. To alleviate this bias a function of DBTBH was added to model 3 forming bark model 6 (Table 1). The term is constructed so that the relative bark thickness is still predicted as one at breast height, but the bark thickness near the tip goes to the inverse of the value of the DBTBH in inches.

Segmented polynomials (upper stem)

Segmented polynomials were first used for tree taper by Max and Burkhart (1976) who cited earlier work by Fuller (1969), Gallant and Fuller (1973), and Gallant (1974). Two or more polynomials are fitted to sections of the range and grafted together at one or more join points. At each join point the model is required to be continuous, and the first derivative must be continuous. Further, Max and Burkhart use the polynomial presented for tree taper by Kozak, Monro, and Smith (1969), and they constrain the model to predict the value 0 at the tip of the tree (relative height equal to 1) .

Maguire and Hann (1990) start with this basic definition and impose the further requirement that the model reflect the tree-to-tree variability in the height to diameter ratio. Maguire and Hann's model is represented in Table 2 by models 7[4] and 8, for relative height and relative diameter, respectively. They applied their model to Douglas-fir, and their model estimates will be examined here using both their parameter estimates and estimates from the data at hand.

Models 9 and 10 are two- and three-segmented polynomials, respectively, on relative diameter with the formulation directly from the work of Max and Burkhart (1976).

Stump models

A common approach to estimating bark thickness below breast height is to use one of the models for the upper stem. An exception is a stump model developed by Maguire and Hann (1990) for predicting bark thickness between breast height and a one foot stump. However, the model by Maguire and Hann was not included in this analysis because it requires height to crown base as one of the predictor variables.

Here we examine four models for predicting bark thickness at the stump. As with the upper stem models, we assume that the bark thickness at breast height can be measured and the variable predicted is the relative bark thickness, $Y = \frac{DBT}{DBTBH}$. The four stump models considered are given in Table 3. They include a simple proportional model (Grosenbaugh's option 1), a

[4] Note that this is the only model considered that uses relative height instead of relative diameter.

hyperbolic model (Grosenbaugh's option 2), a simple linear model, and a power model that reflects the bark thickness at breast height. All models are constrained to predict the relative bark thickness to be equal to 1 when the relative diameter is equal to 1.

DATA

The bark taper data used here are taken from 5 separate sources where bark measurements along the tree stem were made as part of separate studies on tree taper or utilization. Each of these data sources in described briefly here and the number of trees in each data set is shown by species in Table 4.

The "Coop" bark data set comes from a stem analysis data set developed by members of the Northern California Forest Yield Cooperative. Formed in 1978, this cooperative research group links the research staff at the University of California with various private forestry companies and public agencies to produce growth prediction models for California conifers. A previous study (Biging 1984) used the inside-bark tree diameters to develop stem taper equations for California conifers. Measurements on these same trees were also used for initial growth models for CACTOS, the California Conifer Timber Output Simulator (Wensel , Meerschaert and Biging, 1987). The data came from industry forestlands in the Sierra Nevadas, Southern Cascades, the Shasta-Trinity, and the Mendocino region.

The "Dolph" data set comes from a study of conifer trees in California's Sierra Nevadas and was collected for the purpose of developing a California PROGNOSIS variant (Dolph 1984 and 1989). The "Fir Study" data set was contributed to this study by Bill Oliver (USFS, Redding) and was the basis for his growth studies on thinned true fir stands. "Region 6" refers to U.S. Forest Service data from southern Oregon . These data were collected from sample trees in southern Oregon to adjust the regional gross volume inventory estimates for local utilization.

The "Mill Study" data come from a series of utilization studies conducted in California by the USDA Forest Service Pacific Northwest Forest Experiment Station. Sample trees were felled and scaled in the field, scaled again in the mill before being sent through the mill for conversion.

Data screening / editing

The data were received from the various contributors and cooperators and stored in a common data base. The data were then retrieved from this data base using a screening process that

eliminated trees and/or observations that were obviously in error or would result from forked, broken, or grossly malformed trees. Since field data sheets for most of the data were not readily available, there were no further changes made to the edited data.

The mean, minimum, and maximum tree DBH and total height by species are given in Table 5.

RESULTS and DISCUSSION

The SAS nonlinear fitting procedure, NLIN, was used with the Marquardt option (SAS Institute, 1985). Each of the 14 models was fitted to each of the six major conifer species considered. Tables 6 and 7 give the coefficients for the models listed in Tables 1 and 2 for the hyperbolic and segmented polynomial models, respectively. Tables 8 and 9 show the residual sums of squares by species, model, and data set.

Hyperbolic models (upper stem)

For the single parameter models, a distinct improvement was made by fitting the parameter b_2 to the data rather than using the value $b_2 = 2$ as in model 1. A substantial improvement in the quality of the fit was achieved by using the 4-parameter model, model 3. Sums of squares dropped about 21% overall, and a better fit of relative bark thickness was achieved both at the top and the bottom of the tree. The relationship between the single and 4-parameter models is illustrated in Figures 3 and 4 for ponderosa pine and Douglas-fir, respectively. These figures show that the general form of the models 2 and 3 are similar for ponderosa pine but differ considerably for Douglas-fir. The differences in the residual patterns are illustrated in Figures 5 through 8[5].

Figures 5 and 6 for ponderosa pine and Douglas-fir, respectively, show a more balanced residual pattern for the 4-parameter model, model 3. The effect is even more dramatic for Douglas-fir. Model 2 residuals (Figure 7) indicate considerable bias in the estimates in the tips of the trees which appears to be absent in the model 3 residuals (Figure 8). Overestimates of bark in the tips of the trees will have little effect on the estimates of volume in these sections. Its significance is more

[5] Searle (1988) points out that the single or parallel lines with slope -1 in residual plots "occur no matter what model is fitted to y, and correspondingly no matter how $\hat{y}$ is calculated, be it based on linear or nonlinear estimates." Thus the occurrence of such lines here is to be expected and should be of no concern to the reader.

important in the estimation of merchantable height of the tree, i.e., the height at which the tree diameter inside bark is equal to the merchantable limit (say, 4 or 6 inches).

The 4-parameter model fits better than the one-parameter model. However, it may be over-specified. The four coefficients were not all significant for any of the 6 species. This suggests that we should be able to define the relationship with fewer coefficients. Model 4 with b_3 and b_4 dropped and model 5 with only b_3 dropped showed only nominal increases in the residual sums of squares for most species, and in some cases, there were slight decreases in the mean square error (MSE). In summary, for white fir, incense-cedar, and ponderosa pine, there was little difference between the sums of squares for models 3, 4, and 5. The other three species showed considerable model improvement from model 5 to 3.

As evident in Figures 5 and 6, even the 4-parameter model does not do an adequate job of producing zero average residuals in the top of the tree. This pattern was evident in all 6 species. As a result, the hyperbolic model was "adjusted" with a second additive term (see model 6 in Table 1) that was a function of the bark thickness at breast height. Figure 9 shows the form of model 6 for bark thickness 0.5, 1, 2, and 4 inches using coefficients fitted for Douglas-fir. The location of the minimum point for each value of DBTBH is indicated by the line crossing the prediction lines[6]. The relationship between these minimum values and DBTBH is:

$$R_{min} = \frac{1}{a * DBTBH}$$

where R_{min} is the calculated minimum relative diameter for each species as a function of DBTBH and a is a species-specific coefficient. The R_{min} values calculated for the six species appear in Table 10.

This gives a conditional function Y_6' as follows

$$Y_6' = \begin{cases} f_{min} & : \quad RD < R_{min} \\ Y_6 & : \quad RD \geq R_{min} \end{cases}$$

where RD = relative diameter , $\left(\frac{DOB}{DBH}\right)$; R_{min} = diameter ratio at which the minimum bark ratio occurs; and f_{min} = minimum bark ratio (i.e., $Y_6 = f_{min}$ for $RD = R_{min}$).

Model 6 with two coefficients for the hyperbola and one coefficient for the "adjustment" reduced the total sum of squares an average of 8% over all species. A comparison of Figures 11

[6] The minimum points were found by calculating the first derivative of model 6 for each species, setting the derivative to zero, and solving for the range of DBTBH's.

and 12 with Figures 5 and 6, respectively, shows that ponderosa pine with its thicker bark produces more of an "adjustment" in the tops of the trees than does Douglas-fir. However, its real benefit comes from the improvement in the fit in the tops of the trees, as illustrated in Figures 11 and 12, since even the 4-parameter hyperbolic model failed to provide an unbiased model for the top of the tree.

Segmented polynomials (upper stem)

Maguire and Hann (1990) base their 2-sectioned model on relative height. Their model for Douglas-fir, model 7, is shown in Figure 13 with both their coefficients and with coefficients computed from our data. There is little difference between the two models but both show an irregular pattern of bark change as one progresses up the tree.

In an attempt to improve this pattern, we used relative diameter instead of relative height to produce model 8. This change produced a 19% drop in the residual sums of squares over all species with the change being similar for all species. This causes us to reject model 7 in favor of model 8.

Fitting the join point, k, along with the other coefficients produced nearly identical residual sums of squares for all species except white and red firs and even here the improvement in the residual sums of squares was less than 1%. As a result, the value $k = 0.3$ proposed for stem taper by Maguire and Hann (1990) was used in model 8 for all species.

To evaluate the effect of using Maguire and Hann's process for using the height/diameter ratio over a simple 2-sectioned segmented polynomial, we can compare models 8 and 9. Here we see that this step produces no real improvement in the residual sums of squares (Table 9) for ponderosa pine, sugar pine, Douglas-fir, and white fir, but there is a small (though insignificant) improvement for both red fir and incense-cedar. The difference between models 8 and 9 was the greatest for red fir and is illustrated in Figure 14.

Adding a third section to the segmented polynomial model has the potential of improving the fit in the top of the tree. Figures 15 and 16 show the differences between models 9 and 10 for ponderosa pine and Douglas-fir, respectively. There is some slight improvement in the overall residual sums of squares for some species, but, as shown in the residual plots in Figures 17 and 18, any real advantage of using three sections comes from the improvement in the residual pattern in the tips of the trees.

Best upper stem model

Among the segmented polynomial models, the 3-sectioned model, model 10, appears to have an advantage of small residual sums of squares for all species except red fir and incense-cedar and a better residual pattern for all species. Comparing the best hyperbolic model, model 6, with the best segmented polynomial model, model 10, shows that model 6 has the smallest overall residual sums of squares and mean squared error. While, as shown for ponderosa pine and Douglas-fir in Figures 19 and 20, the lines are somewhat similar. In addition to having the smallest residual sums of squares, model 6 only requires that 3 coefficients be computed (compared to 6 coefficients for model 10).

Stump model

Table 12 compares the residual sum of squares (RSS) of the four stump models tested. The model S4 consistently had a much smaller RSS compared to the other three models. The general hyperbolic model, S2, did quite poorly. Residuals from models S1, S2, and S3 all show significant correlation with DBTBH. The models S1, S2, and S3 all showed significant correlation with DBTBH or DBH. The addition of the exponent as a function of DBTBH in model S4 eliminated this correlation and reduced the overall correlation of predicted with residual relative bark ratios.

This extensive look at conifer bark taper has resulted in the recommendation of two new models for bark taper. For upper stems the adjusted hyperbolic model, Y_6', is recommended and for stump bark taper the power model S4 is recommended. Both represent distinct improvements in the way bark taper estimates have been handled in previous studies.

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Table 1. Hyperbolic bark taper equations evaluated, where Y is the predicted bark ratio $\frac{DBT}{DBTBH}$. Double bark thickness is then predicted by multiplying Y by DBTBH.

No. 1 Hyperbolic ratio (Grosenbaugh 1974, option 2)

$$Y_1 = \left(\frac{DOB}{DBH}\right)\left(\frac{1}{2 - \left(\frac{DOB}{DBH}\right)}\right)$$

No. 2 No. 1 with coefficient fitted to data (Brickell 1970, option 2)

$$Y_2 = \left(\frac{DOB}{DBH}\right)\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)$$

No. 3 Brickell option 3

$$Y_3 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)^{b_3}}\right)^{b_4}$$

No. 4 Brickell option 4

$$Y_4 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)$$

No. 5 Variant between Brickell's options 3 and 4

$$Y_5 = \left(\frac{DOB}{DBH}\right)^{b_1}\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)}\right)^{b_4}$$

No. 6 Adjusted hyperbolic ratio

$$Y_6 = \left(\frac{DOB}{DBH}\right)\left(\frac{b_2 - 1}{b_2 - \left(\frac{DOB}{DBH}\right)^{b_3}}\right) - \left(\frac{\left(\frac{DOB}{DBH}\right)^{b_5} - 1}{DBTBH}\right)$$

for DOB > D_{min} and $Y_6 = F_{min}$ otherwise where D_{min} and F_{min} denote the coordinates of the minimum of th above function.

Table 2. Segmented polynomial bark taper equations evaluated, where Y is the predicted bark ratio $\frac{DBT}{DBTBH}$. Double bark thickness is then predicted by multiplying Y by DBTBH.

No. 7 Localized two-segmented polynomial on relative height (Maguire and Hann 1990)

Let $X = \frac{HT}{THT}$ and

$$a_1 = b_1 + b_2 \left(\frac{THT}{DBH}\right) + b3 \left(\frac{THT}{DBH}\right)^2$$

then

$$Y_7 = 1 + z_1 + a_1 z_2 + a_2 z_3$$

for

$$z_1 = I \left\{ \frac{(X - 1)}{k - 1} \left[1 + \frac{(k - X)}{k - 1} \right] - 1 \right\}$$

$$z_2 = X + I \left\{ \frac{(X - 1)}{k - 1} \left[X + \frac{k (k - X)}{k - 1} \right] - X \right\}$$

$$z_3 = X^2 + I \left\{ \frac{k (X - 1)}{k - 1} \left[2 X - k + \frac{k (k - X)}{k - 1} \right] - X^2 \right\}$$

with $k = 0.3$ and $I = 1$ for $X \geq k$ and zero otherwise. The coefficients a_2, b_1, b_2 and b_3 to be estimated.

No. 8 Localized two-segmented polynomial on relative diameter

Model as above except $X = 1 - \frac{DOB}{DBH}$.

No. 9 Two-segmented polynomial on relative diameter (Max and Burkhart 1976).

$$Y_9 = b_1 (X - 1) + b_2 (X^2 - 1) + b_3 (a_1 - X)^2 I_1$$

where $X = 1 - \frac{DOB}{DBH}$, $I_1 = 1$ for $X \leq a_1$ (zero otherwise), and a_1, b_1, b_2 and b_3 are coefficients to be estimated.

No. 10 Three-segmented polynomial on relative diameter (Max and Burkhart 1976).

$$Y_{10} = Y_9 + b_4 (a_2 - X)^2 I_2$$

where Y_9 is defined above, $I_2 = 1$ for $X \leq a_2$ (zero otherwise) and a_2 and b_4 are additional coefficients.

Table 3. Models considered for predicting the relative bark thickness at stump, Y, where DOB is the diameter outside bark at the stump and DBHOB is the stump.

Grosenbaugh's (1974) bark option 1

$$Y_{S1} = \frac{DOB}{DBH} \qquad (S1)$$

Grosenbaugh's (1974) bark option 2[5] :

$$Y_{S2} = \left(\frac{DOB}{DBH}\right)\left(\frac{1}{2 - \left(\frac{DOB}{DBH}\right)}\right) \qquad (S2)$$

A simple linear model:

$$Y_{S3} = 1 + c_1 \left(\frac{DOB}{DBH} - 1\right) \qquad (S3)$$

where c_1 is a coefficient .

The above linear model expanded by a power computed from the bark thickness at breast height:

$$Y_{S4} = 1 + c_1 \left(\frac{DOB}{DBH} - 1\right)^{(c_2 + c_3\, DBTBH)} \qquad (S4)$$

where c_1, c_2, and c_3 are coefficients estimated from the data.

[5] See footnote number 3

Table 4: Number of trees by Species and Datasets:

SPECIES & DATASET	*NO. OF TREES*
Ponderosa Pine	
COOP	197
DOLPH	141
MILL	177
ALL	515
Sugar Pine	
COOP	65
DOLPH	62
MILL	54
ALL	181
Douglas-Fir	
COOP	186
DOLPH	44
ALL	230
White Fir	
COOP	352
DOLPH	313
MILL	161
FIR STUDY	76
ALL	902
Red Fir	
COOP	36
MILL	148
FIR STUDY	562
REGION 6	273
ALL	1019
Incense Cedar	
COOP	81
DOLPH	189
MILL	54
ALL	324

Table 5. Number of trees, mean, minimum, and maximum diameter at breast height and total height used to fit bark taper models by species.

		Diameter			Height		
SPECIES	Number of trees	Mean	Min	Max	Mean	Min	Max
Ponderosa pine	515	21.1	5.5	60.2	100.7	18.0	202.0
Sugar pine	181	24.1	3.5	62.5	94.8	9.0	204.0
Douglas-fir	230	15.4	5.5	31.2	86.7	25.5	150.6
White fir	902	17.6	5.5	62.9	80.2	18.6	204.0
Red fir	1019	20.9	3.0	57.4	85.6	10.7	195.0
Incense-cedar	324	13.8	3.4	50.8	49.5	7.4	155.0

Table 6. Coefficients calculated for each hyperbola model.

SPECIES	MODEL #	b_1	b_2	b_3	b_4	b_5
Ponderosa Pine	2		2.05347*			
	3	.172435*	1.00677*	.029665*	.649123*	
	4	.095474	1.31635			
	5	.349935	1.09045		.388315	
	6		2.85323	6.18840		.150480
Sugar Pine	2		3.35449			
	3	-.021007*	1.01705*	.031161*	1.10561*	
	4	.219662	1.46440			
	5	.679999	60.7655*		43.2687*	
	6		4.93961	443.812*		.098117
Douglas-Fir	2		1.65893			
	3	.721122	8.01472*	11.3072*	3.89232*	
	4	.193696	1.27256			
	5	.614854	1.00847		.150581	
	6		2.83504	13.6250		.055268
White Fir	2		1.79431			
	3	.186981*	.989607*	-.062291*	.572898*	
	4	.077587	1.28022			
	5	.250571	1.14080		.575405	
	6		2.67181	5.82073		.090351
Red Fir	2		3.46315			
	3	.256430	.944620	-.450786	.313750	
	4	.177066	1.48455			
	5	.641516	42.6754*		31.7995*	
	6		6.16610	52.9442		.095993
Incense Cedar	2		2.43416			
	3	.200062*	.996129*	-.012814*	.715154*	
	4	.381861	1.46319			
	5	.751977	1.00961		.105277	
	6		3.60876	9.34989		.060086

Table 7. Coefficients calculated for each segmented polynomial model.

SPECIES	MODEL #	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Ponderosa Pine	7	-2.83552	-.205678	.017598	5.91209				
	8	-2.68503	-.133710	.011663	4.50627				
	9	.352847				.173569*	-.649957	4.10060	
	10	.174750	.921401			49.8355*	-27.2028*	7.66930	27.5576*
Sugar Pine	7	-2.68484	-.185314	.015244	5.56146				
	8	-2.39568	-.080486	006466	3 75759				
	9	.235572				-.164997	-.501945	5.78076	
	10	.039684	.938852			73 5727*	-39 4965*	115.620*	39 5929*
Douglas-Fir	7	-4.34707	.249995	-.012264	5 48281				
	8	-3 94069	.085076	- 002958	5 56035				
	9	.215982				-.155540	-.396021	9.47537	
	10	.185933	.948954			38.3031	-20.5246	10.8073	20.4324
White Fir	7	-3.11532	-.077110	.010508	5 30685				
	8	-2.81758	-.143934	.011142	4 85128				
	9	.386240				387050	-.757016	4.13236	
	10	.226295	.923448			52 2488	-28.4194	5.21484	28.7965
Red Fir	7	-2 07456	-.140579	.009701	4.23880				
	8	-1.73861	-.222968	.015331	3.34266				
	9	.206937				-.298995	-.433534	6.05364	
	10	.124518	.941632			55.9460	-30 0884	11.7623	30.0820
Incense Cedar	7	-3 42176	-.163537	.024326	6.03113				
	8	-1.96023	-.293174	.027806	3.47811				
	9	.242376				-.451194	-.235204	5.24302	
	10	214742	.968996			87.0817	-45.3019	5.56182	45.2831

Table 8. Residual Sum of Squares (RSS) for the different Bark Models by Species and Datasets:

HYPERBOLIC MODELS

SPECIES & DATASET	NO. OF OBS.	Model# 1	Model# 2	Model# 3	Model# 4	Model# 5	Model# 6
Ponderosa Pine							
COOP	1232	42.40	42.10	33.30	33.80	33.30	29.70
DOLPH	836	8.47	8.56	5.54	5.40	5.54	5.05
MILL	673	17.37	17.52	13.91	14.08	13.92	13.07
ALL	2741	68.30	68.22	52.75	53.27	52.73	47.86
Sugar Pine							
COOP	424	18.41	14.81	12.53	12.49	13.27	11.02
DOLPH	351	2.53	4.12	2.56	2.73	4.59	3.00
MILL	242	9.89	8.55	7.55	7.65	8.27	6.40
ALL	1017	30.83	27.49	22.64	22.87	26.13	20.95
Douglas-Fir							
COOP	1164	27.30	26.50	18.90	20.70	18.90	17.60
DOLPH	237	3.32	2.65	1.47	1.43	1.48	1.41
ALL	1401	30.70	29.14	20.37	22.16	20.41	19.04
White Fir							
COOP	2048	64.10	66.20	49.60	50.29	49.60	45.40
DOLPH	1988	24.90	21.90	15.50	15.22	15.50	15.40
MILL	514	11.69	11.34	9.87	9.88	9.87	10.26
FIR STUDY	441	6.70	6.05	4.12	4.12	4.11	3.88
ALL	4991	107.40	105.48	79.09	79.51	79.08	74.91
Red Fir							
COOP	178	5.53	4.64	3.34	3.63	4.07	3.23
MILL	452	12.80	9.90	9.10	9.20	9.70	9.15
FIR STUDY	3607	68.80	64.00	44.00	46.80	60.60	42.50
REGION 6	1367	43.80	31.70	29.40	30.40	29.60	30.51
ALL	5604	130.60	110.35	85.84	90.03	104.04	85.38
Incense Cedar							
COOP	355	6.65	6.14	5.66	5.76	5.66	5.26
DOLPH	844	7.89	8.48	6.03	6.16	6.03	4.91
MILL	114	6.78	5.79	5.41	5.69	5.42	5.72
ALL	1313	21.32	20.41	17.10	17.61	17.11	15.89

Table 9. Residual Sum of Squares (RSS) for the different Bark Models by Species and Datasets.

SEGMENTED POLYNOMIAL MODELS

SPECIES & DATASET	NO. OF OBS.	Model# 7	Model# 8	Model# 9	Model# 10
Ponderosa Pine					
COOP	1232	38.30	34.70	34.70	33.10
DOLPH	836	11.44	5.61	5.60	5.63
MILL	673	17.78	14.10	14.31	13.88
ALL	2741	67.50	54.40	54.63	52.61
Sugar Pine					
COOP	424	14.46	12.90	12.77	11.67
DOLPH	351	6.92	2.88	2.93	3.73
MILL	242	7.48	7.11	7.14	7.07
ALL	1017	28.87	22.88	22.84	22.46
Douglas-Fir					
COOP	1164	19.40	19.4	19.3	19.1
DOLPH	237	3.36	1.39	1.43	1.38
ALL	1401	22.75	20.75	20.76	20.48
White Fir					
COOP	2048	59.2	52.4	52.0	49.8
DOLPH	1988	27.3	16.2	15.9	15.6
MILL	514	11.55	9.37	9.89	9.83
FIR STUDY	441	4.66	4.19	4.20	4.12
ALL	4991	102.70	82.07	81.99	79.29
Red Fir					
COOP	178	3.96	3.57	3.49	3.30
MILL	452	9.72	9.10	9.2	9.1
FIR STUDY	3607	48.99	44.60	45.6	44.5
REGION 6	1367	37.07	28.28	29.3	29.2
ALL	5604	99.76	85.56	87.59	86.07
Incense Cedar					
COOP	355	6.99	5.83	5.70	5.64
DOLPH	844	12.93	5.75	6.10	6.06
MILL	114	6.17	4.94	5.33	5.35
ALL	1313	26.10	16.51	17.13	17.05

Table 10. Coefficient values for calculation of Rmin for California conifers.

SPECIES	a
Ponderosa pine	9.241
Sugar pine	9.241
Douglas-fir	15.078
White fir	10.098
Red fir	13.388
Incense cedar	15.883

Table 11. Coefficients calculated for stump bark models.

SPECIES	Model #	b_1	b_2	b_3
Ponderosa Pine	S3	1.99610		
	S4	1.73416	0.0*	0.475413
Sugar Pine	S3	0.813498		
	S4	1.0*	0.0*	0.678624
Douglas-Fir	S3	1.95171		
	S4	1.0*	0.0*	0.397823
White Fir	S3	1.19022		
	S4	1.0*	0.0*	0.555289
Red Fir	S3	1.23417		
	S4	0.880132	0.260076	0.225097
Incense Cedar	S3	2.23461		
	S4	1.87591	0.559340	0.171258

* fixed

Table 12. Residual sum of squares (RSS) for the different stump bark models by species.

SPECIES	No. of Obs.	Model# S1	Model# S2	Model# S3	Model# S4
Ponderosa Pine	462	124.37	114.86	107.18	85.18
Sugar Pine	128	13.60	28.25	13.80	10.83
Douglas-Fir	189	32.90	32.21	29.06	21.92
White Fir	856	83.48	180.26	82.36	70.81
Red Fir	1002	68.33	223.06	65.41	54.99
Incense Cedar	300	84.63	74.38	48.69	46.05

Figure 1. Illustration of 3 bark options (G1, G2, and G3) provided by STX (Grosenbaugh 1974)

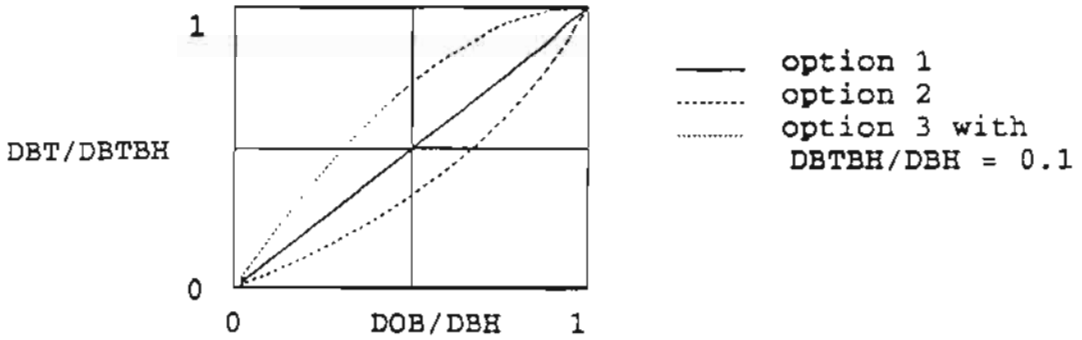

Figure 2. Bark ratio for measurements on 352 white fir trees with Grosenbaugh's bark option 2 for reference.

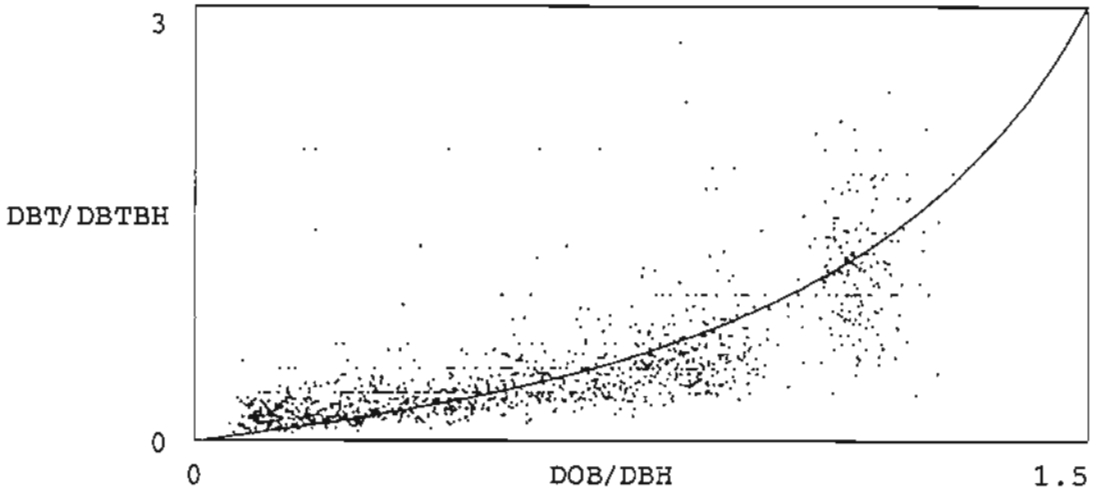

Figure 3. Single parameter (Y2) vs. 4 parameter hyperbola (Y3) for Ponderosa pine

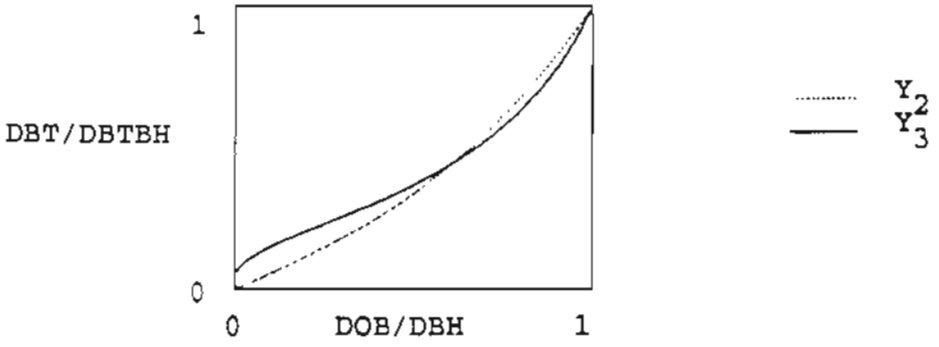

Figure 4. Single parameter (Y2) vs. 4 parameter hyperbola (Y3) for Douglas-fir

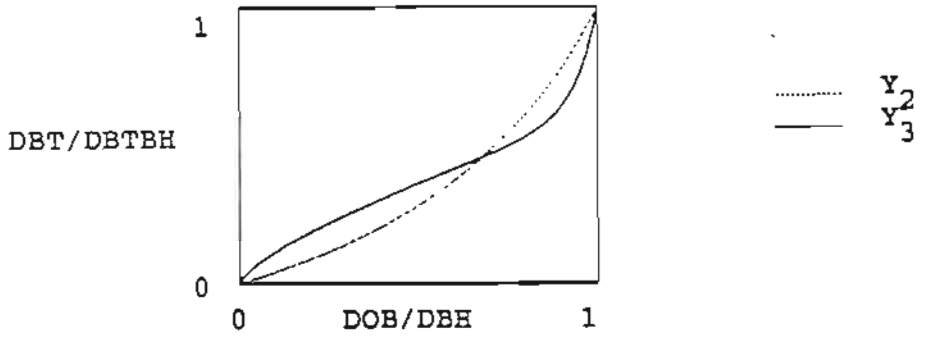

Figure 5. Residual (observed-predicted) plots for 4-parameter hyperbola (Y3) bark ratio for Douglas-fir.

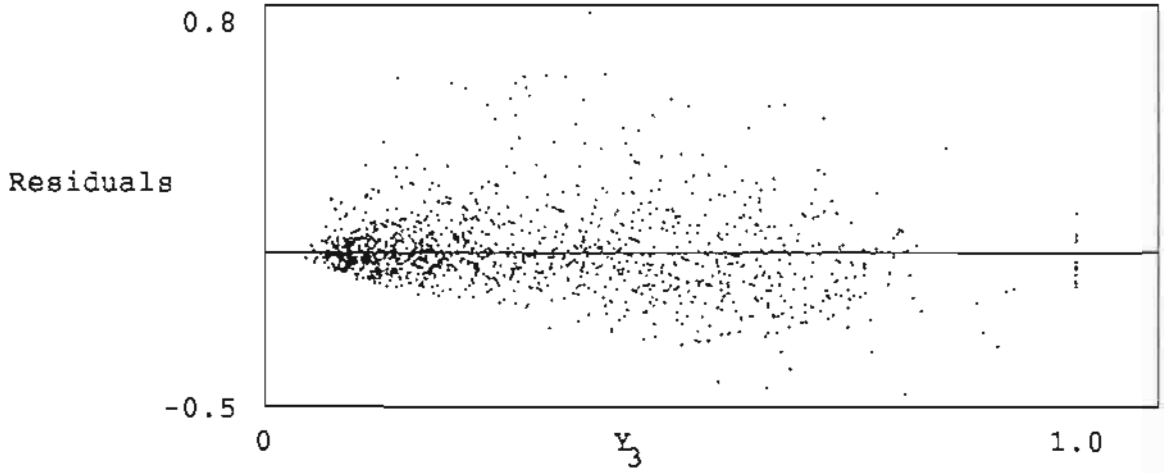

Figure 6. Residual (observed-predicted) plots for 4-parameter hyperbola (Y3) bark ratio for Ponderosa Pine.

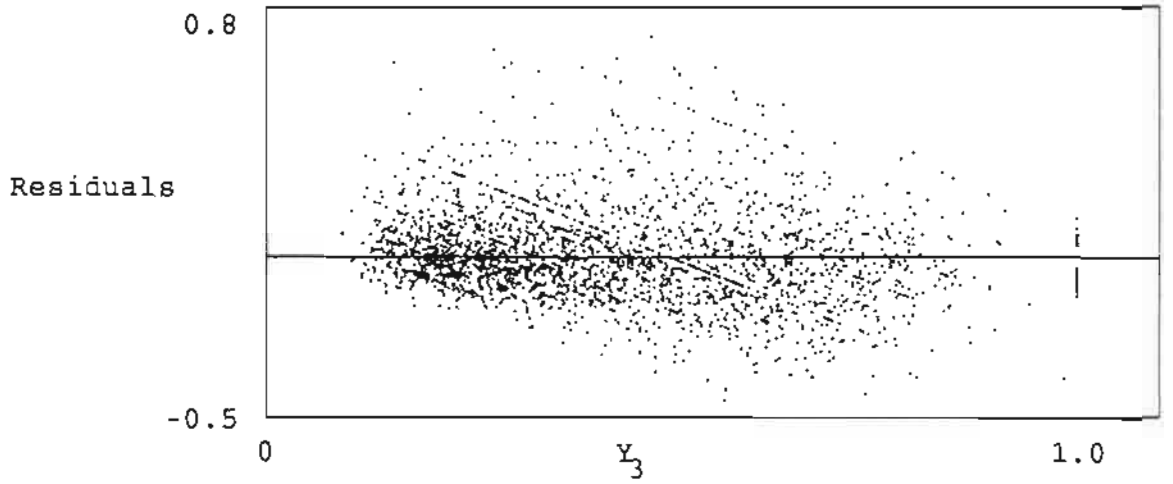

Figure 7. Residual (observed-predicted) plots for single-parameter hyperbola (Y2) bark ratios for Douglas-fir.

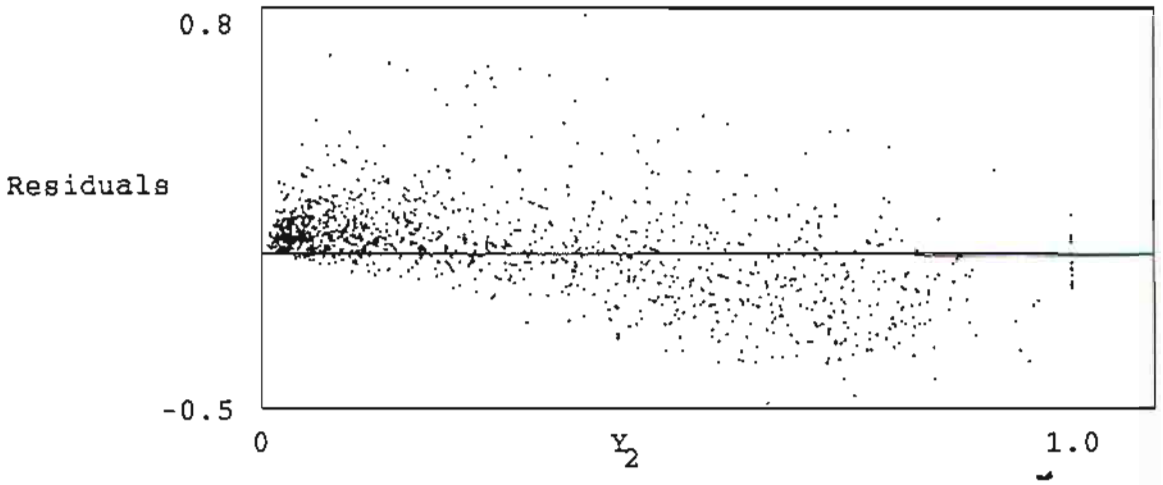

Figure 8. Residual (observed-predicted) plots for single-parameter hyperbola (Y2) bark ratio for Ponderosa Pine.

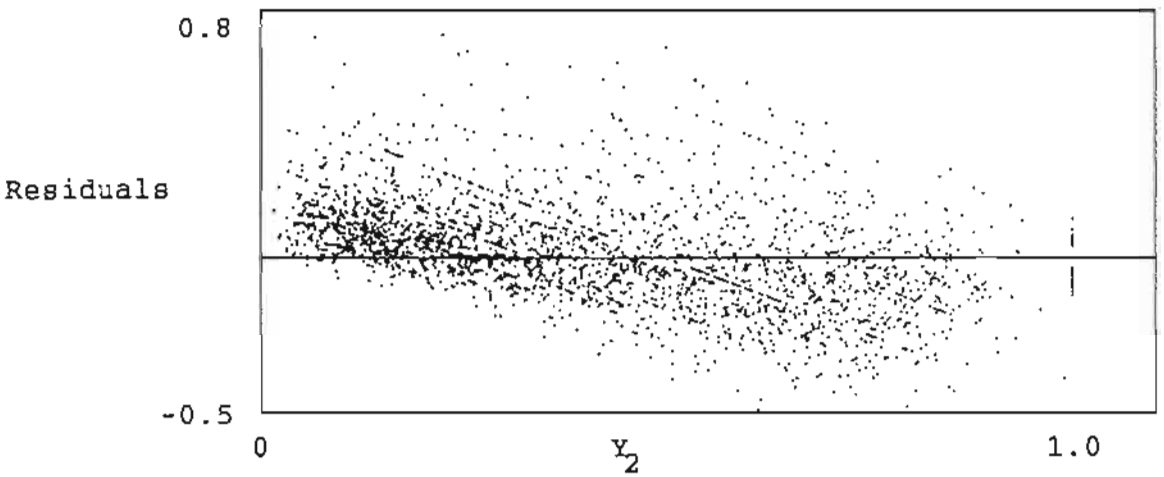

Figure 9. Adjusted hyperbola (Y6) for Douglas-fir (DBTBH = .5, 1, 2, and 4 inches).

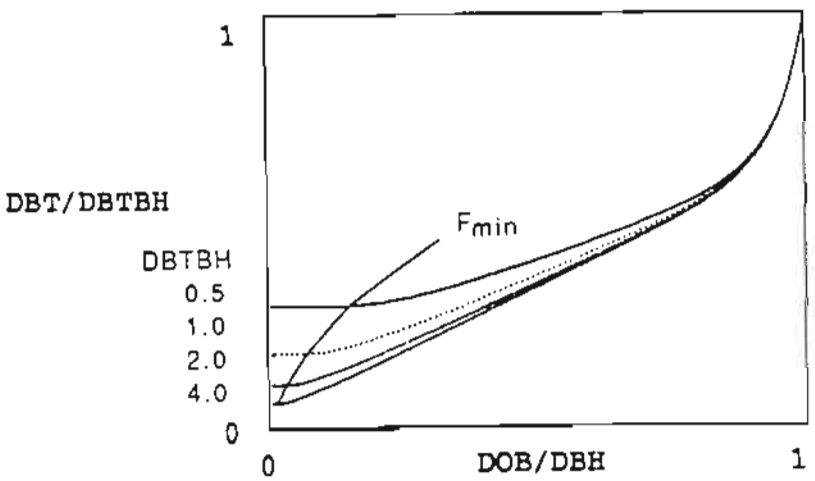

Figure 10. Adjusted hyperbola (Y6) vs. 4-parameter hyperbola (Y3) for Douglas-fir (DBTBH = 2.3 inches).

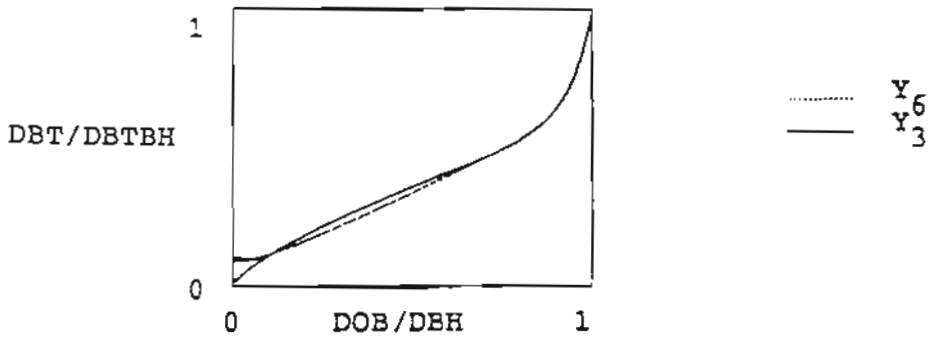

Figure 11. Residual (observed-predicted) plots for adjusted hyperbola (Y6) bark ratios for Ponderosa pine.

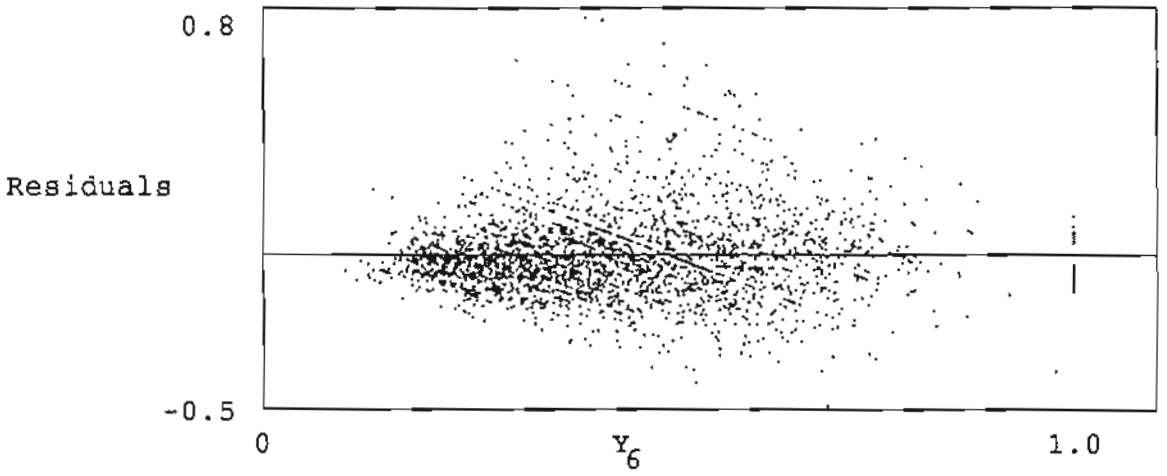

Figure 12. Residual (observed-predicted) plots for adjusted hyperbola (Y6) bark ratios for Douglas-fir.

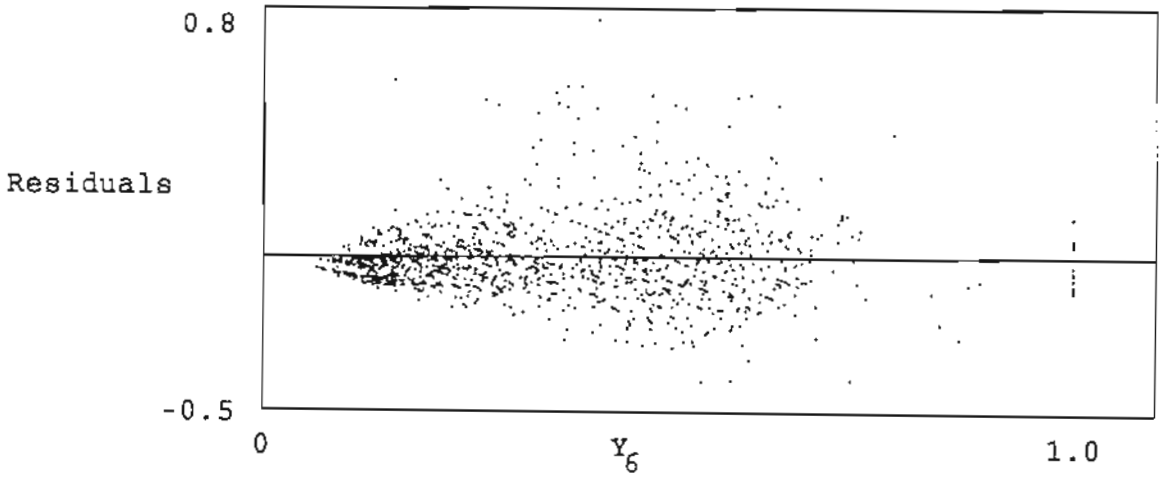

Figure 13. Illustration of the relative height model using Maguire and Hann's coefficients (Y7m) and using new coefficients (Y7) for Douglas-fir.

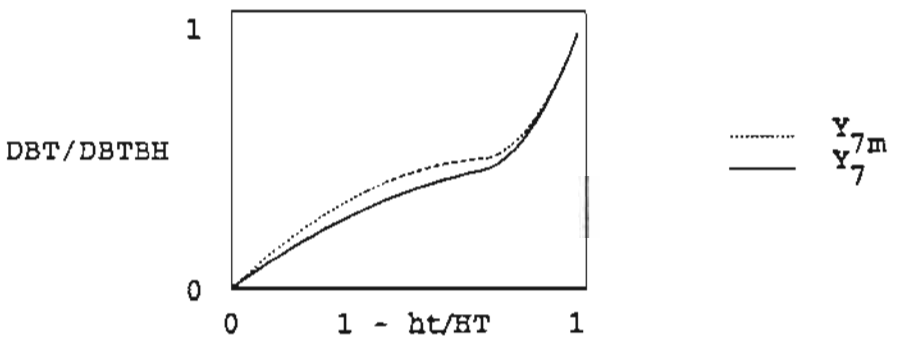

Figure 14. Maguire and Hann's model (Y8) on relative diameter vs. two-segmented polynomial (Y9) for Red Fir.

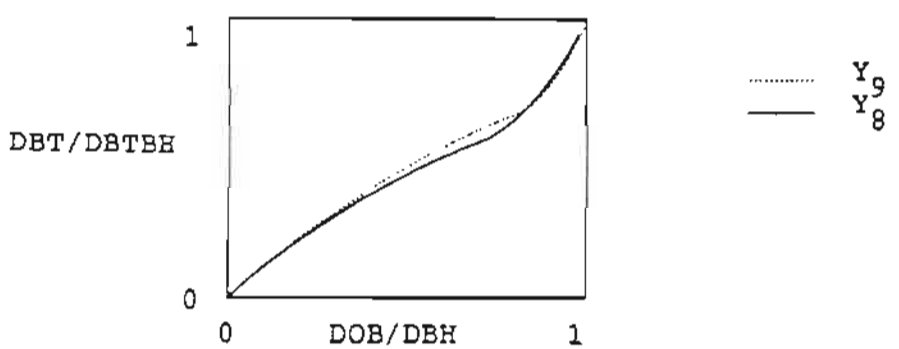

Figure 15. Two-segmented polynomial (Y9) on relative diameter vs. three-segmented polynomial (Y10) for Ponderosa pine.

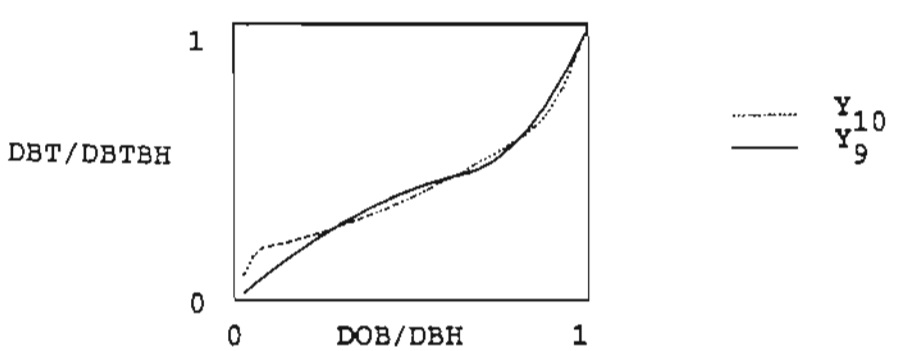

Figure 16. Two-segmented polynomial (Y9) on relative diameter vs. three-segmented polynomial (Y10) for Douglas-fir.

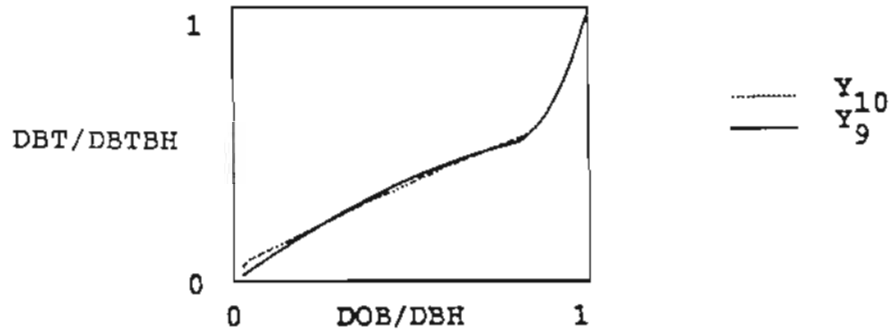

Figure 17. Residual vs. predicted bark ratios 2-segmented polynomial (Y9) for Douglas-fir.

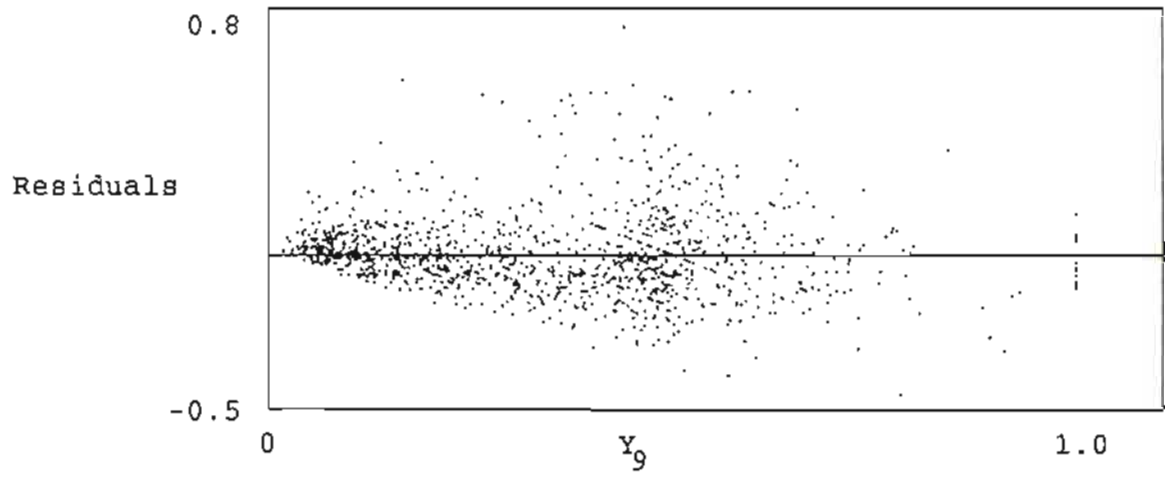

Figure 18. Residual vs. predicted bark ratios 3-segmented polynomial (Y10) for Douglas-fir.

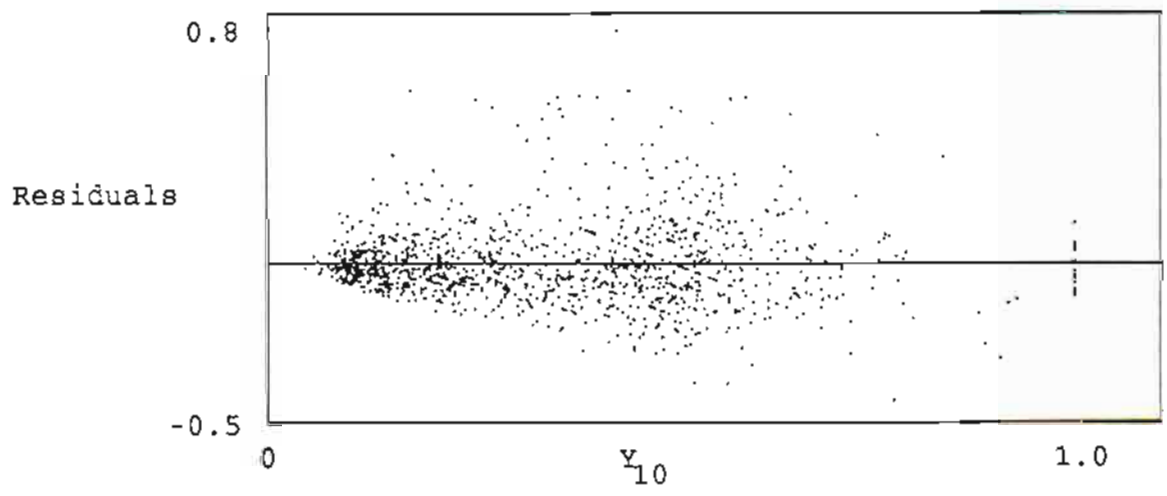

Figure 19. Adjusted hyperbola (Y6) vs. three-segmented polynomial (Y10) for ponderosa pine (DBTBH = 2.3 inches).

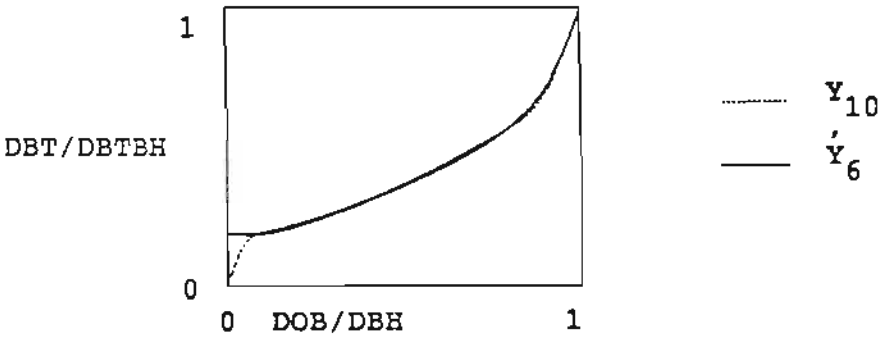

Figure 20. Adjusted hyperbola (Y6) vs. three-segmented polynomial (Y10) for Douglas-fir (DBTBH = 2.3 inches).

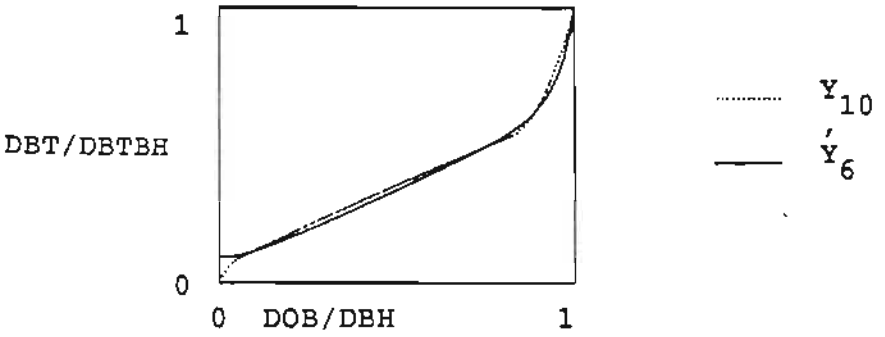

Figure S1. Residuals vs. predicted bark ratio for ponderosa pine using stump model 1.

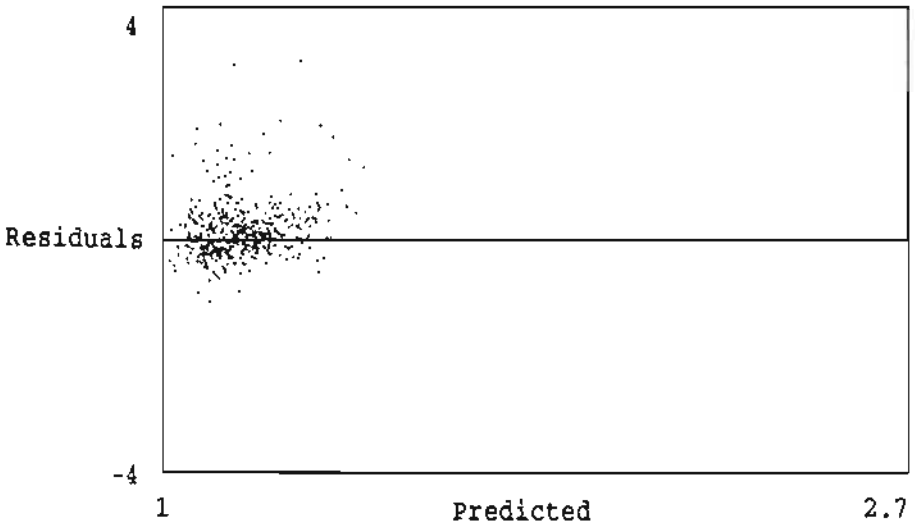

Figure S2. Residuals vs. predicted bark ratio for ponderosa pine using stump model 2.

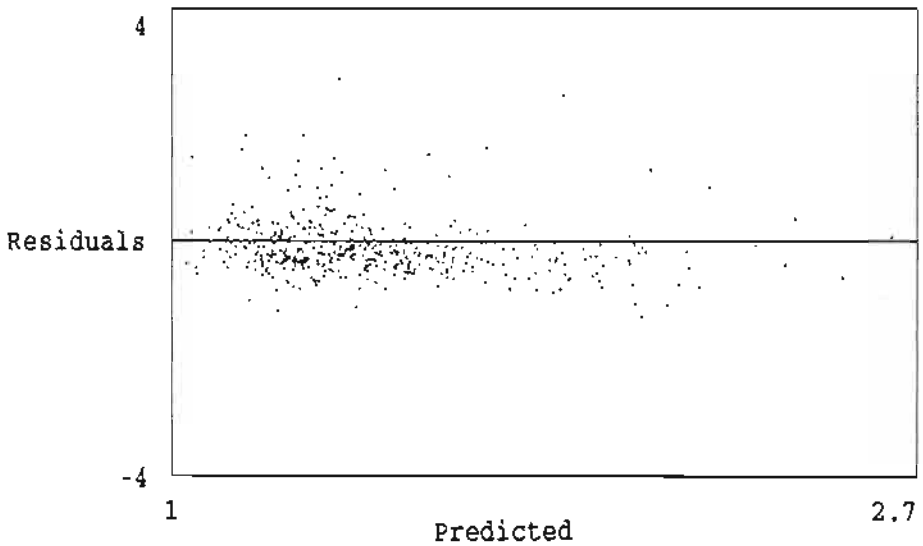

Figure S3. Residuals vs. predicted bark ratio for ponderosa pine using stump model 3.

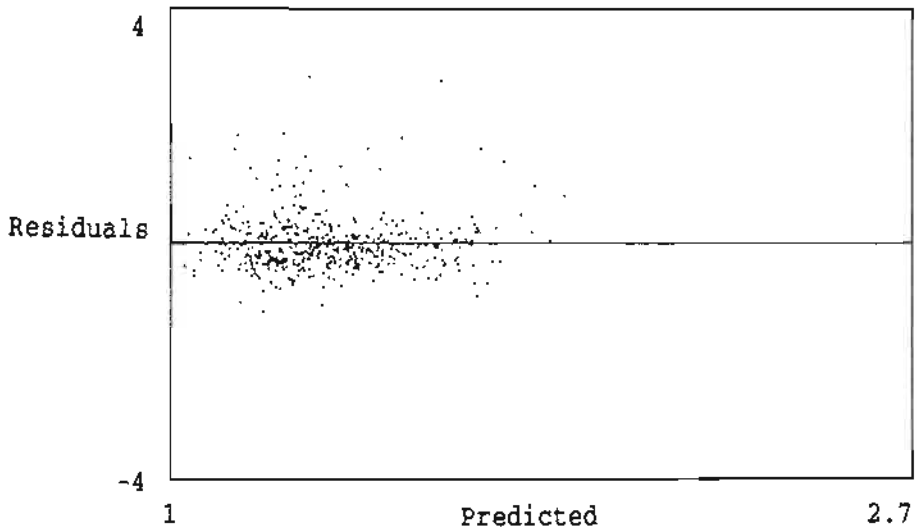
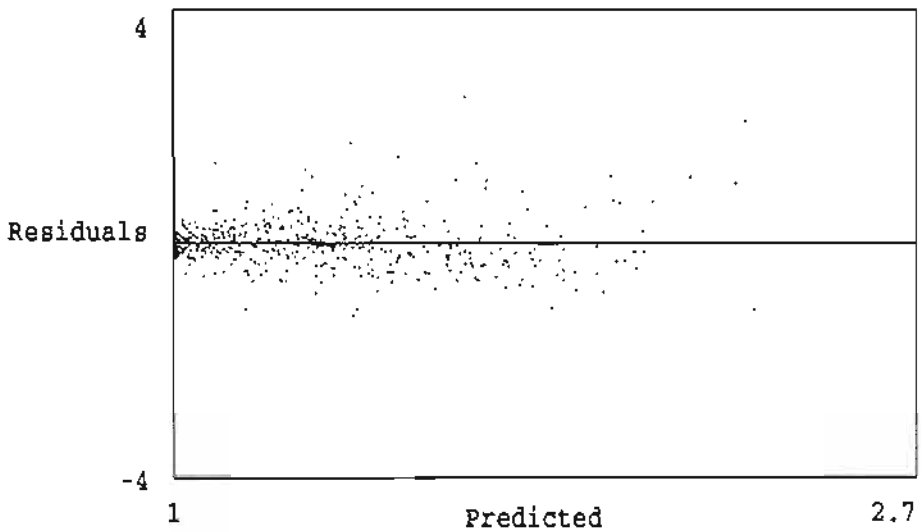
Figure S4. Residuals vs. predicted bark ratio for ponderosa pine using stump model 4.